

Naturally Time Dependent Cosmological Constant in String Theory

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Abstract

In the light of the proposal of Ref. [1], we discuss in detail the issue of the cosmological constant, explaining how can string theory naturally predict the value which is experimentally observed, without low-energy supersymmetry.

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1 Introduction

One of the most puzzling problems of modern theoretical physics is the explanation of the existence of a non-vanishing “vacuum energy” in the Universe. In the recent years the non-vanishing of this quantity has been experimentally confirmed [2, 3]. The cosmological constant has been measured with a certain degree of accuracy, and it turns out to be $\sim 10^{-122}$ in M_{P}^2 units, of the order of the squared inverse age of the Universe ($\sim 10^{61} M_{\text{P}}^{-1}$). The fact that the value of a physical cosmological parameter of the dimension of an energy squared, and therefore an inverse time squared, turns out to be precisely given by the inverse square of the age of the Universe, induces to suspect that it could be a manifestation of quantum mechanics at a cosmological level. A scenario able to justify this value should then by consistency In such a framework, the value of the cosmological constant should in fact correspond to the minimal energy fluctuation “out of the vacuum” allowed for a quantum system aged as our Universe, and it should find its explanation in a quantum gravity framework. This however means that we should consider the space-time itself as an “entity” extended up to the present horizon, whose existence already by itself constitutes a quantum system, even before taking into account possible matter and/or radiation inside it. From this point of view, the empty space-time is not “empty”, even when any contribution of matter and radiation has been subtracted. This concept of vacuum state is natural in string theory, basically a “quantum theory of the space-time (coordinates)”. Indeed, string theory provides a configuration of space-time (the target space coordinates) and, for any configuration, possesses a vacuum state with a definite energy. However, if, once the string configuration has been specified, the computation of the vacuum energy is in principle unambiguous, what seems to be not so natural is to explain in a string framework the actual value of the cosmological constant, i.e. to produce the “right” string configuration. In particular, what appears very hard is to couple a small value of this parameter with a heavy supersymmetry breaking. In this note, we will discuss how the present experimental value of the cosmological constant can be obtained in a natural way from string theory, if the latter

is provided with an appropriate interpretation of the target space coordinates. The point of view we are going to consider is the one of Ref [1], that we will now explain more in detail. The usual way string theory is interpreted is in terms of a map from world sheet to target space coordinates, in which the latter describe points in the space-time. The specific point of this map inside the space-time has no special meaning: space-time is infinitely extended and possesses no privileged point to be considered as the absolute “origin” of a physical frame better than any other one. This means that the system possesses invariance under space and time translations. As discussed in Ref. [1], one may wonder whether this is actually an unavoidable assumption, or whether with this we are requiring too much from our theory. After all, what we can observe at any time is the Universe just up to the horizon. Objects placed beyond the horizon can not influence our present observations, and there is no deep ground to require, for a theory supposed to describe the world we observe, to know, at any given time, also about what happens outside our horizon. At any time the amount of space we can observe is bounded by the horizon set by the distance covered by light since the origin of the Universe, and therefore has a radius given, in units of the speed of light, by the age of the Universe. For such a theory, at any given time the space is then compact.

The hypothesis we make is that the string target space coordinates describe this kind of “compactification”. However, unlike an ordinary compactification of coordinates, e.g. onto circles, here the compact space *is*, at any time, *the space*. This means that it has not to be thought of as something embedded into something else. What is infinite in our case is the span, the range of values, of the string coordinates, which a priori are allowed to take, during the history of the Universe, values in an infinitely extended four-dimensional space. The physical space-time is however always finite, and there is no invariance under translation of space and/or time coordinates, because in such a space it is not immaterial the location of a point, with respect to the boundary of the space. The translation of a coordinate, $x \equiv (x_0, x_1, x_2, x_3) \rightarrow x + a$, amounts to going closer or farther from the horizon, and this is not a symmetry. Moreover, there is no embedding space-time in which to translate the full system. The coordinates are therefore “absolute”. We talk in this case of “absolute space-time”.

2 The cosmological constant

We will now discuss how, in this new interpretation, the present-day small value of the cosmological constant, corresponding to the inverse of the (squared) Hubble constant, arises as a consequence of the absence of low-energy supersymmetry. The first consideration in order is: what do we have to intend for the cosmological constant? Historically introduced as a “constant” term in the Einstein’s action, it can be shown to correspond to the energy of the “vacuum”. There is some arbitrariness in the interpretation of this number, in the sense that an analogous contribution to this effective parameter may be provided, through the energy-momentum tensor term, by matter and fields. Therefore, it is possible to think that this term receives contribution from “dark” energy or matter. What we consider here is the pure *energy of the vacuum*, namely, the energy of space-time itself, once the contribution of any kind of matter and radiation has been subtracted. Therefore, there is no “microscopical”

field description of processes contributing to this quantity: this is a pure quantum effect. As such, the “total” value of the cosmological constant comes from the string computation of the vacuum energy. Differently from what happens in field theory, in string theory the “ground” value is unambiguously determined, once the particular “string vacuum” is specified. Normally, in a situation of supersymmetry broken at the string/Planck scale, the value produced by string theory is too large, unless one introduces some mechanism enabling to lower the string scale, or to make the value of the cosmological constant independent of it, such as the existence of low-energy supersymmetry in some sector of the theory (e.g. gravity sector) in which experiments have been so far less sensitive. The situation we want to consider is however the one of an intrinsically non-perturbative “M-theory” vacuum, with supersymmetry “hardly” broken at the Planck scale, as described in Ref. [1]. Namely, the mass gap Δm between particles, fields, and their supersymmetric partners (including the mass difference between graviton and gravitino) is of order one in Planck units, and the coupling of the theory in four dimensions is also of order one, as the volume of the internal coordinates. Naively, one would say that, unless miraculous non-perturbative cancellations occur, these conditions lead to a cosmological constant of order one. We will however see that in our scenario this is not the case. The crucial point is how do we compare the string calculation with the “constant” parameter in the Einstein’s effective action. Let’s for the moment consider the case of a perturbative string vacuum: how to deal with the non-perturbative scenario corresponding to the actual situation of interest for us will be discussed later. Traditionally, parameters computed on the string side (as is eq. 2.5), are to be compared with the integrand appearing in the effective action. In other words, as it is usually defined, the string calculation computes “densities”, values per unit of volume. The reason is that, in an infinitely extended space-time, there is a “gauge” freedom, corresponding to the invariance under space-time translations. There is therefore a redundancy in any calculation, related to the fact that any quantity computed at the point “ \vec{x} ” is the same as at the point “ $\vec{x} + \vec{a}$ ”. In order to get rid of the “over-counting” due to this symmetry, one normalizes the computations by “fixing the gauge”, i.e. dividing by the volume of the “orbit” of this symmetry \equiv the volume of the space-time itself. Actually, since it is not possible to perform computations with a strictly infinite space-time, multiplying and dividing by infinity being a meaningless operation, the result is normally obtained through a procedure of “regularization” of the infinite. Namely, one works with a space-time of volume V , supposed very big but anyway finite, and then takes the limit $V \rightarrow \infty$. In this kind of regularization, the volume of the space of translations is assumed to be V , and it is precisely the division by V what at the end tells us that we have computed a density. In any such computation this normalization is implicitly assumed. Notice also that, in this approach, it is also implicitly assumed that, even for finite volume, T-duality of the space-time coordinates remains broken, something necessary in order to be allowed to speak of an effective action, for a universe of “large” volume.

In our case however, we do not assume the invariance under translations to be preserved for compact space-time, and indeed it is not: the situation we are describing is not the one of a “compactification”: for us space-time is “absolute”, is extended up to the “horizon”, which sets a volume that, with some simplification, can be thought to be represented through a certain boundary value of the string target coordinates, \vec{X} , identifying a “radius” of com-

pactification. A translation of a point inside this space, $\vec{x} \rightarrow \vec{x} + \vec{a}$ is not a symmetry, being \vec{X} fixed. On the other hand, a translation of the boundary value of the target space string coordinate \vec{X} represents an evolution of the Universe, not a symmetry of the present-day effective theory: there is no “outside” space in which the coordinate \vec{X} is embedded ¹! In this framework, the volume of the group of translations is not V : simply this symmetry does not exist at all. There is therefore no over-counting, and what we compute is not a density, but a global value. In other words, what in the traditional interpretation is a density, the value of a quantity at a certain point of space-time of the present-day Universe, in our case turns out to be a density in the “space” of the whole history of the Universe, the (global) value of such quantity at a certain point of its history. In the specific case of the cosmological constant, we have that the usual dependence of the cosmological constant density on the points \vec{x} of space-time gets promoted to a dependence on the points labeling the history of the Universe, \vec{X} : $\Lambda(\vec{x}) \rightarrow \Lambda(\vec{X})$. In order to see this, we will have to discuss some subtleties related to the notion of effective action, something we will do in a moment. Notice that, in this interpretation of string coordinates, there is no “good” limit $V \rightarrow \infty$, if for “ ∞ ” we intend the ordinary situation in which there is invariance under translations. This symmetry appears in fact only strictly at the limit. The volume of translations is some kind of “delta-function” supported at infinity. On the other hand, this is not a problem in our picture: infinite space-time does not really belong to the history of the “universe” ², for which the horizon, although increasing, will always be finite.

2.1 On the side of the effective action

In this set up, it is not a priori obvious that one can even speak of an effective action to compare with: in order for this to have any meaning, T-duality along the space-time coordinates must be broken. Only in this case it is possible to speak of “large” Universe and “expansion”, as something distinguished from the “contraction”. The scenario we have in mind is therefore the one depicted in Ref. [1], in which T-duality is indeed broken, and it is possible to talk about an effective action, in which the light modes of the theory, namely those whose mass is below the Planck scale, move in a space-time frame of coordinates larger than the Planck length. In this picture, as usual heavier modes are integrated out and their existence manifests itself only through their contribution to the parameters of the effective theory. Such a description is possible at any time of the evolution of the Universe after the breaking of T-duality along the space-time coordinates, a condition we will from now on implicitly assume to be realized. At any point in the evolution, labeled by a boundary value

¹In principle, one could think to translate all the points of the “segment” $[\vec{0}, \vec{X}]$, namely, also the origin. In this case, a translation would be a symmetry, and we would recover the usual result. The point is that, in order to translate also the origin, which in our set up is the origin of the history of the Universe, and of “time” itself, we would need an “embedding history”, a “space-time”, and in particular a “time”, larger than, and containing, our one, existing therefore before and after it. But this is precisely what we have excluded in our basic hypothesis. For our theory, space-time *is* the one we observe.

²Namely, of the “observed universe”, the one that matters for the theory.

“ \vec{X} ” in the string space of above, there is therefore an effective action for the light modes:

$$S_{\vec{X}} = \int_{[\vec{0}, \vec{X}]} d^4x (R + \Lambda + \dots) , \quad (2.1)$$

where the integration region is a “ball” centered on the observer and bounded by the horizon. The r.h.s. of eq. 2.1 has been converted to the Einstein frame, where lengths are measured in Planck units³. This is what in our set up substitutes the traditional effective action. For what matters the local physics there is no much difference, as the contribution of the boundary can safely be neglected at our time of “very large” Universe. When instead we want to look at “cosmological” phenomena, such as the cosmological constant, or the cosmological evolution of couplings, masses and, in general, the “fundamental constants”, the boundary enters heavily in the game. Let’s concentrate on the second term of the r.h.s. of 2.1, namely:

$$S_{\vec{X}}[\Lambda] \equiv \int_{[\vec{0}, \vec{X}]} \Lambda d^4x . \quad (2.2)$$

On the side of the effective action, the integration is performed on a domain extended as the Universe up to the present-day horizon. The effective action of local physics is therefore obtained by extrapolating the local physics, the one valid at the point where the observer is located, also to points located at a non-vanishing distance in space, and time, from the observer. This is possible, because we know that the Universe is extended (at least) up to the present-day horizon, and therefore we can think at this “world” at any time t up to the present age of the Universe, that we indicate with \mathcal{T} , although we cannot observe it all simultaneously⁴. On the effective theory side we have therefore a space-density of cosmological constant $\Lambda(t)$ such that:

$$S_{\vec{X}}[\Lambda] \approx \int_{[\vec{0}, \vec{X}]} \Lambda(t) d^4x . \quad (2.3)$$

The value to be compared with the string output is therefore:

$$\lambda(t = \mathcal{T}) \equiv \int_{[\vec{0}, \vec{X}]_{t=\mathcal{T}}} \Lambda(t = \mathcal{T}) d^3x \approx \mathcal{T}^3 \Lambda(\mathcal{T}) . \quad (2.4)$$

2.2 On the string side

Let’s consider the computation of the “vacuum energy” in a string vacuum of interest for our problem. Indeed, the physical situation we have in mind, the one described in Ref. [1], corresponds to a non-perturbative vacuum, in which the internal coordinates are of the order of the Planck scale. As we there observed, even when all the (M-theory) internal coordinates, including the one playing the role of coupling, are at the Planck scale, and

³To the purpose of this discussion, we can safely consider a flat metric: $\sqrt{-g} = 1$. The corrections provided by matter and the cosmological constant itself are small in Planck units, and are therefore of second order.

⁴The time t runs effectively, in Planck units, in the interval $1 \leq t \leq \mathcal{T}$.

therefore we are in a strong coupling regime, it is nevertheless possible, whenever there are extended space coordinates, to map to a (dual) perturbative string vacuum in lower dimensions. The latter is obtained by trading one space coordinate for the internal coordinate playing the role of (higher dimensional) coupling⁵. With good approximation the value of the vacuum energy is therefore obtained with a one-loop computation. Since we “integrate over” all space coordinates, the value obtained in this way is not the cosmological constant of a lower dimensional theory: it is indeed the quantity we are looking for. The reason is that the quantity we compute is not a density but a global value, referred to a given point in the history of space-time, no matter of the role played by the coordinates (whether coupling, or internal, or “external” space coordinates) in the specific string approach we are considering. Thanks to this passage through a dual picture, we are therefore able to deal with the problem of the cosmological constant in a string vacuum with coupling of order one, and a (four dimensional) string scale of the same order as the Planck scale. The details of the dual realization of the actual physical configuration are irrelevant to the purpose of the present discussion: it is sufficient to know that *all* the ground mass gaps are of order one to conclude that the result is of order one. In order to understand this, consider for concreteness a “generic” closed string, heterotic-like realization of the string configuration. The results we want to extract are however to be thought in a string duality-invariant perspective, for which the choice of a specific vacuum is just a matter of convenience. The “cosmological term” reads then:

$$S_{\bar{X}}[\Lambda] \sim \mathcal{Z} \approx \int_{\mathcal{F}} \sum_i (-1)^Q q^{P_L^{(i)}} \bar{q}^{P_R^{(i)}}, \quad (2.5)$$

where \mathcal{Z} is the one-loop partition function. As usual, $q = e^{2\pi i\tau}$, \mathcal{F} indicates the fundamental domain of integration of the torus complex coordinate τ and Q indicates the supersymmetry charge. The string computation tells us that, under the condition of mass gap between target-space “bosons” and “fermions” of order one in Planck units, $\Delta P \sim 1$, in the *light-cone gauge* the partition function is of order one:

$$\mathcal{Z} \sim \mathcal{O}(1). \quad (2.6)$$

In the infinite volume case, when translation and reparametrization invariance is unbroken, the value obtained in this way can be directly compared with the density parameter appearing in the integrand of the effective action, eq. 2.2, to conclude that $\Lambda \sim \mathcal{O}(1)$. In our case,

⁵If the (target) space of the theory was given in terms of a simple factorization of coordinates, this procedure would be as simple as rigorous. In practice, however, we have no reason to expect such a naive situation, and, as discussed in Ref. [1], there are good reasons to expect that this is not the case. If the external space is not factorized out, but it is in some way “embedded” in the whole space, that looks more like a “fibration”, it is not possible to simply trade one external coordinate for the coupling of a lower dimensional theory: the coupling cannot be singled out. However, as long as we are interested just in the order of magnitude of the cosmological constant, we do not expect the corrections due to the embedding of the coordinates into a more complicated geometry to be so dramatic, and the above arguments are legitimate. For analogous reasons, in the following we will consider the string living in a flat background. Strictly this is not the physical case, precisely because the non-vanishing cosmological constant induces a curvature. However, as we already pointed out in the footnote 3, such corrections to the flat background are of the order of the cosmological constant. Therefore, they contribute only at the second order to the determination of this quantity, and can be neglected for the purposes of the present discussion.

however, besides the space-time volume factor we loose owing to the missing translational invariance, we have also to take into account the fact that now also reparametrization is not anymore an invariance of our theory. This means that, although this remains a basic invariance of the underlying string construction, the comparison with the effective action must take into account the real “state” of the coordinates under which a certain result has been obtained. The map between world-sheet and target-space coordinates is forcedly degenerate, and the reparametrization invariance tells us that we have the freedom to rescale coordinates in such a way to, roughly speaking, “identify” two target-space coordinates with the world-sheet ones. String amplitudes are then obtained by integrating over these coordinates up to their “horizon”, in this case corresponding to the string world-sheet size. This means that we have shrunk a dimension-two subspace of space-time to the string proper size. The real effective action to compare with is therefore the one in which space-time has also been shrunk in this way. We can adjust for this “asymmetry” of space-time by simply switching on an inverse Jacobian for this rescaling of coordinates on the string side, thereby multiplying the string result by a space-time “sub-volume” V_2 . This volume is basically obtained by squaring the Jacobian corresponding to the boost of the time interval from the string proper length to the age of the Universe. If x_0 is the world sheet variable along the world sheet time coordinate, running in an interval of length ℓ_S , the target space time coordinate is $t \sim \mathcal{T} \times x_0/\ell_S$, where \mathcal{T} is the age of the Universe. The Jacobian under consideration is $|\partial t/\partial x_0|$, and we obtain:

$$\lambda(t)|_{(string\ frame)} = \left| \frac{\partial t}{\partial x_0} \right|^2 \times \mathcal{Z} \sim \mathcal{T}_S^2 \times \mathcal{Z}, \quad (2.7)$$

where \mathcal{T}_S is the age of the Universe in string units. The final step of any string computation, required in order to compare with the effective action, is to convert the computation performed in the string frame into string-string duality invariant variables, i.e. to the Einstein frame. In this frame, the proper length is not the one of the string, ℓ_S , but corresponds to the Planck scale, $\ell_P \equiv 1/M_P$ in $c = \hbar = 1$ units. Expression (2.7) is given in units of the string mass scale, $M_S \equiv 1/\ell_S$. However, for what we said, despite the passage through a perturbative vacuum, this expression accounts for the actual value of a four-dimensional theory with coupling of order one, in which therefore the string scale can be considered equivalent to the Planck scale, $\ell_S \equiv \ell_P$. The Jacobian $\left| \frac{\partial t}{\partial \ell_0} \right|$ turns out therefore to be the age of the Universe \mathcal{T} , as measured in Planck units: $\mathcal{T}_S \approx \mathcal{T}$. As a result, we obtain⁶:

$$\lambda(t)|_{string} \sim \frac{1}{\mathcal{T}} \times \mathcal{T}^2 \times \mathcal{Z} \approx \frac{1}{\mathcal{T}} \times \mathcal{T}^2 \times \mathcal{O}(1). \quad (2.8)$$

The factor \mathcal{T}^2 is precisely the two-volume factor V_2 we have just discussed. The factor $1/\mathcal{T}$ comes instead from the reintroduction of a normalization of the string computation to match with the fact that on the effective action we did not integrate over the time coordinate,

⁶Notice that the integration over the world-sheet coordinate $\tau \equiv x_0 + ix_1$, implied by the one-loop computation of \mathcal{Z} responsible for this result, does not correspond to an integration over the history of the Universe: this would instead correspond to an integration over the values of the Jacobian, $|\partial t/\partial x_0| = \mathcal{T}$.

therefore we don't have to compare with an integral value along the time. The comparison of the string output with the effective action gives therefore:

$$\lambda(t = \mathcal{T}) \approx \mathcal{T}^3 \times \Lambda(\mathcal{T}) \approx \mathcal{T} \times \mathcal{O}(1), \quad (2.9)$$

where the second term comes from the integral of the effective action over the space coordinates, eq. 2.4, while the third term is the string result. From 2.9 one obtains immediately:

$$\Lambda(\mathcal{T}) \sim \frac{1}{\mathcal{T}^2} \approx 10^{-122} \text{M}_{\text{P}}^2. \quad (2.10)$$

We stress that it is precisely the “built in” failure of invariance under time translations, implied by our starting point of considering space-time up to the horizon as the “whole” space-time of the theory, what is responsible for the time dependence of this parameter. This is a key point of our set up. It is precisely this change of perspective what allows us to obtain time-dependent parameters without time-dependent vacuum expectation values of fields. In our framework, not only the cosmological constant naturally acquires a time dependence, but also curvature terms, and, as we saw in Ref. [1], masses and couplings. With the interpretation of the string space-time compactifications as describing “points” in the history of the Universe, the lack of space-time translations invariance is then precisely equivalent to the statement of time dependence of “fundamental constants”.

3 Comments

In the previous section we have discussed the comparison between string theory and a parameter of the effective action, the cosmological “constant”, in which the later was allowed to carry an explicit dependence on time. If one takes the traditional approach and considers the space-time infinitely extended, then the cosmological evolution must be explicitly introduced in the effective action, and this is the only choice. If instead the domain of integration is supposed to be bound by the horizon, as in our case, it is also possible to take a different point of view, and consider the parameters of the effective action as constant inside the domain of integration. The two approaches turn out to be equivalent, because in the second case the time dependence is anyway introduced, through the dependence on time of the boundary, namely of the distance of the horizon from the observer. This means that, at any time of the evolution of the Universe, we will have to compare string theory with a different effective action, differing not only in the extension of the domain of integration but also in the value of the effective parameters. This second approach is in some sense more “democratic”, in what it does not artificially single out the evolution only along the coordinate we call “time”.

Indeed, when the horizon is finite both approaches are equivalent, because they differ by a volume factor of the time, corresponding to the integration one has to perform, in the second case, also over this coordinate, in order to compare with the effective action corresponding to the actual configuration of the Universe. This volume factor cancels against an analogous factor present in the definition of densities. In the first case, where the time dependence of the cosmological constant is explicit, in order to compare with the time-dependent parameter

of the effective action we don't have to integrate over the time coordinate; there is on the other hand also no translation invariance along the time coordinate, and therefore we also miss a volume factor of this coordinate in the normalization of the density. In order to see the equivalence of the two approaches, let's see here how the comparison goes in the second case, namely when we don't introduce an explicit time dependence in the cosmological constant. In this case eq. 2.3 becomes:

$$S_{\vec{X}}[\Lambda] \approx \int_{[\vec{0}, \vec{X}]} \Lambda(\mathcal{T}) d^4x, \quad (3.1)$$

where $d^4x = dt dx_1 dx_2 dx_3$. Notice that, in the integrand on the r.h.s., Λ does not depend on the “bulk” time variable t but on the boundary value \mathcal{T} . It is therefore a constant in the domain of integration. We have on the other hand a series of different effective actions, one for each point of the evolution of the “boundary” coordinate \mathcal{T} , each one with a different parameter Λ . \mathcal{T} can therefore be considered here as a parameter labeling the various effective actions, one per each size of the horizon.

As opposite to the previous case, now what we have to compare with the string result is the global value of this effective cosmological term, i.e. the full expression 3.1, which includes also the integration over the time coordinate t . Instead of expression 2.4, we have here:

$$\int \lambda(t = \mathcal{T}) dt = \int_{[0, \vec{X}]} \Lambda(\mathcal{T}) d^4x \approx \mathcal{T}^4 \Lambda(\mathcal{T}). \quad (3.2)$$

In this case in fact the effective theory is invariant under translation of the “time” coordinate t running inside the region of integration $[0, \mathcal{T}]$ (being Λ constant, in order to well define translations also close to the boundary, we can think to identify the points 0 and \mathcal{T} , and introduce periodic boundary conditions). The comparison with the string computation performed in the light-cone gauge gives then:

$$S_{\vec{X}}[\Lambda] \approx \int_0^1 d^2x \int_0^{\mathcal{T}} d^2x \Lambda(\mathcal{T}) = \mathcal{T}^2 \times \Lambda(\mathcal{T}) \Leftrightarrow \mathcal{O}(1), \quad (3.3)$$

where now we obtain a factor \mathcal{T}^2 and not just \mathcal{T} . On the right of the symbol \Leftrightarrow we quote the string output. By switching on the Jacobian \mathcal{T}^2 , accounting for the transformation to the real size of the present day coordinates, we recover the conditions of the output obtained from the effective action, 3.2, with all the four space-time coordinates extended:

$$S_{\vec{X}}[\Lambda] \approx \int_0^{\mathcal{T}} d^4x \Lambda(\mathcal{T}) = \mathcal{T}^4 \times \Lambda(\mathcal{T}) \Leftrightarrow \mathcal{T}^2 \times \mathcal{O}(1). \quad (3.4)$$

The result is in any case the same: $\Lambda(\mathcal{T}) \sim 1/\mathcal{T}^2$.

4 Conclusions

In this work we have discussed how a change of point of view allows to justify in a natural way the observed value of the cosmological constant in string theory. One of the key points

of our set up, proposed in Ref. [1], is that the “effective” space-time of the theory has, at any point of its history, a finite volume, the one of the region bounded by the horizon of observation. In this framework, there is no invariance under space-time translations: space-time is “absolute”. When considered as a quantum system, this “universe” turns out to possess a minimal energy gap, accounted for by the so called “cosmological constant” (as we discussed in Ref. [1], masses themselves can be interpreted somehow as the effect of the “inertia”, or resistance, of the system to the attempts to change its configuration by displacing an object from a geodesic to another one). In absence of low-energy supersymmetry, the value of the cosmological constant turns out to correspond to the one we would obtain by considering that this parameter is a measure of the (square of the) “energy gap” of the Universe within the time of its existence, as derived from the Heisenberg’s Uncertainty Principle:

$$\Delta E \Delta t \geq 1, \quad (4.1)$$

if we use $\Delta t = \mathcal{T} \equiv$ the age of the Universe. This relation has the following interpretation: since the “time” length of our observable Universe is finite, the vacuum energy cannot vanish: it must have a gap given by (4.1). The cosmological constant, which has the dimension of an energy squared, must then correspond to the square of this quantity. In other words, it originates as a pure quantum effect: since it is not protected by any symmetry, it is naturally generated by “quantum corrections”, and its size is set by the natural cut-off of the Universe. Notice that, in the framework of our proposal, a relation between the value of the cosmological constant and the rate of expansion of the Universe is established at a quantum level. This relation is however based on the fact that both the cosmological “constant” and the rate of expansion depend on the point in the history of the Universe, i.e. on its age. There is no “direct” relation of the cosmological constant to the Hubble parameter, independently on the age of the Universe: this comes rather as a consequence of the fact that the cosmological constant, the rate of expansion of the Universe and the time dependence of masses and couplings are all related to the expansion of the horizon, the latter being a consequence of the existence of massless modes (photon, graviton). What happens is that: 1) the expansion of the horizon generates a time dependence of the cosmological constant and masses; 2) a decrease of masses produces in turn the expansion of the Universe. This phenomenon is basically due to the effective repulsive force felt by matter in a system in which masses decrease with time (Ref. [1]). As we discussed in Ref. [1], the non-homogeneous mass decrease, in particular the different time-scaling of the mass of proton and electron, leads to a shift in the wavelength of emitted radiation corresponding to the one recently observed in ancient quasars (Refs. [4, 5, 6, 7]), an effect sometimes attributed to the “time-dependence of the coupling α ” (also in our scenario the coupling varies with time, but in our case its variation is not the (main) responsible for the shift in the spectra). On the other hand, the change with time of masses and couplings is not in contradiction with the bounds coming from the Oklo natural reactor (Ref. [8]) or from the nucleosynthesis. Both in fact are bounds on cross sections, whose variation depends not just on a single scale but on the overall effect of all the mass and coupling variations. Only under the hypothesis of keeping fixed all scales except from the coupling allows to convert these constraints into bounds on the variation of this coupling. This is not our case.

Giving up with the idea of including infinity within the “regular” configurations of the

system, namely, considering the history of space-time volumes as an open set, implies that there is no non-trivial limit of parameters of the theory, as the volume V of the Universe “inside the horizon” expands toward infinity. As $V \rightarrow \infty$, the parameters of the theory, i.e. couplings, masses, and the cosmological constant, tend either to zero or to infinity (this second is the case of the strong coupling and masses of string states above the Planck scale). A non-trivial value for these parameters is instead obtained in the traditional approach, in which infinity is included in the configurations of the Universe. This introduces a divergence in the (string) theory, whose regularization leads to finite non-trivial limits for these parameters: for instance, for the vacuum energy.

As opposite to the common point of view, here we don’t consider a space-time of finite volume V as a sort of “regularization of infinity”, artificially introduced in order to solve computational problems, while having however in mind the infinite volume limit as the true, physical situation. In our approach, a space-time of finite volume is the “real”, effective physical situation at any finite time. This perspective allows us to address many physical questions in a completely different way. As discussed in Ref. [1], a consequence of the above mentioned “trivialization” of infinity is that now the “geometry” of space-time turns out to evolve toward a “flat” limit. This is in some sense the “classical” limit of the system, an asymptotic configuration never realized in practice. The scenario is therefore quite different from the usual one, in which cosmology is based on a model of the Universe considered as a perfect fluid. Here this description cannot be applied, because the “truncation” of the theory at the horizon prevents from introducing a smooth metric for a space-time extending also beyond it. The kinematic equations for the expansion of the Universe cannot be transferred in a simple way to the present scenario, because in our case the expansion is not adiabatic. Indeed, since the “boundary” of the effective universe expands at the speed of light, its energy is not conserved: if we start at a certain point with a certain matter content, the energy corresponding to this “small universe” will be conserved under expansion, because by definition the horizon itself expands together with radiation (there is no radiation flowing out of the horizon). However, new matter will appear inside the horizon, and energy will not be conserved for this system. This effect is not balanced by a decrease of vacuum energy: the vacuum energy density scales in fact like the square root of the cosmological constant, therefore as the inverse of the age of the Universe, $\rho(E)_{\text{vac}} \sim \frac{1}{T}$, and the total vacuum energy within the horizon increases therefore as the square of the age of the Universe, $E_{\text{vac}} \sim T^2$. The system is therefore somehow holographic, at least for what matters the properties of the vacuum. In this set up, this is on the other hand not the case for the matter content and its energy. However, we have seen in Ref. [1] that matter does not feel an “ordinary” geometry of space-time; there is therefore nothing to worry about the fact that the matter energy density does not fit with a simple geometric volume/boundary relation.

In the interpretation proposed in Ref. [1], the quantization of the string coordinates, among which is our space-time, turns out to be something quite different from the ordinary *local* quantization of fields and particles. We have two “levels” of quantization: the local one, concerning just the neighbor of the observer, for which it is a good approximation to consider space-time as infinitely extended. This leads to the ordinary quantization and an effective action describing a “time-independent” physics. We have then the level of the

quantization of space-time itself, leading to the cosmological evolution of the Universe. As opposite to the traditional approach, what we propose is that string theory does not provide us with an “improved” description of the “local” physics. It gives something else: the evolution of the “bubble” inside which we can define what we call “space-time”, namely the frame in which everything we observe takes place. In this approach, certain parameters of the “local” effective theory, like masses and the cosmological constant, have to be seen as effective terms, whose origin is explained by switching to the “cosmological” description, the “global” description provided by string theory interpreted as we said. These parameters are related to global facts, like the size of the horizon, and therefore depend on the point \vec{X} in the history of the Universe.

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